# Methods for Modelling of Overall Telecommunication Systems

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**Abstract.** The aim of the paper is to summerize some of the methods for modeling of overall telecommunication systems developed by the authors at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences and to propose new methods using the apparatus of the Generalized Nets (GNs) theory. On the basis of the discussed methods, two basic tasks about overall telecommunication systems are formulated. Analytical models for solving the Quality of Service (QoS) prediction task and the Network Dimensioning/Redimensioning Task (NDT/NRDT) are proposed.
A classical model of overall telecommunication system is considered. General teletraffic tasks are formulated on the basis of a proposed conceptual model. Some assumptions for the system are stated which allow for a relatively simple analytical model to be obtained. Analytical expressions for basic teletraffic characteristics of the main tasks about overall telecommunication systems are derived. Graphical representation of the results is included. A comparison with other approaches for network dimensioning is made and is represented graphically.

**Keywords:** Overall telecommunication system · Conceptual modeling · Quality of Service · Overall network dimensioning/redimensioning.

## 1 Introduction

Despite the long-standing use of the telecommunication systems in practice, their modelling is still an object of current development. The aim of the paper is to summarize some of the methods for conceptual and analytical modeling of overall teleocmmunication systems (with and without queuing) developed at the Institute of Mathematics and Informatics (IMI) of the Bulgarian Academy of Sciences (BAS). These methods differ from the traditional widely accepted and often used methods in the literature. The motivation behind the development and the application of new methods is based on the fact that the results of the use of the traditional methods are not always presented with satisfying accuracy.

The proposed and used method for conceptual modeling in Section 2, Section 3 and Section 4 of the present paper is based on an original approach and graphical language, developed at IMI - BAS. The method of the Generalized

Nets (GNs) is used in Section 5 in the conceptual modeling of overall telecommunication system with queuing.

Two main tasks about overall telecommunication system without queuing are formulated and solved analytically. They are:

1. task for prediction of the Quality of Service (QoS)(Section 3);
2. task for dimensioning of networks with QoS guarantees (Section 4).

The numerical results of the application of the proposed methods are presented graphically. A comparison is made between the results of the proposed method and other well known and often used in the practice methods.

The analytical model (Section 5) for determining the important teletraffic parameters of the overall telecommunication system with queuing in the Switching stage, from a practical point of view, is based on the GNs conceptual model and Queuing theory.

## 2 Classical model of overall telecommunication system

The classical model of overall telecommunication system is proposed in [19] and developed in more details in [21]. It is a detailed conceptual traffic model of an overall (virtual) circuit switching telecommunication network, like PSTN and GSM, including users' behaviour, with: BPP (Bernoulli-Poisson-Pascal) input flow; repeated calls; limited number of homogeneous terminals; losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned communication.

The described approach is applicable directly for every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of core and access networks traffic modelling. For packet switching systems, like Internet, proposed approach may be used as a comparison basis.

The traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately but in their interrelation. Two types of virtual devices are included in the model: base and comprising base devices.

### 2.1 Base virtual devices representation and their parameters

At the bottom of the structural model presentation, we consider basic virtual devices that do not contain any other virtual devices. A basic virtual device has a general graphical representation as shown in Fig. 1.

The parameters of the basic virtual device x are the following (see [10] for terms definition):

- $Fx$ - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;

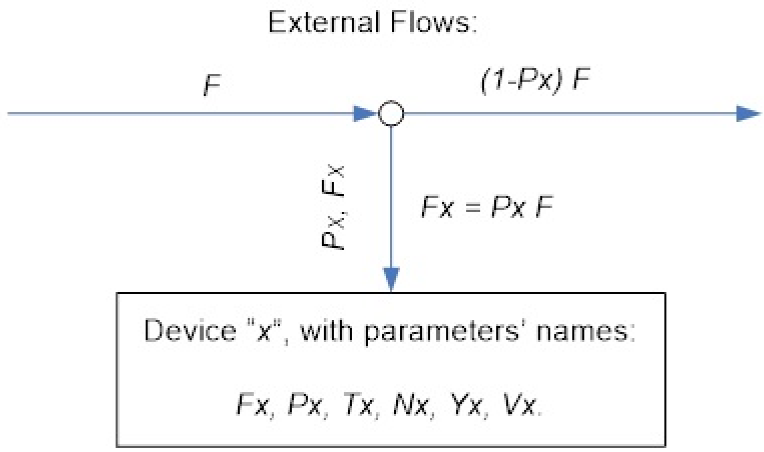


Fig. 1: A graphical representation of a basic virtual device x.

- $Px$ - probability of directing the requests towards device x;
- $Tx$ - service time (duration of servicing of a request) in device x;
- $Yx$ - traffic intensity [Erlang];
- $Vx$ - traffic volume [Erlang - time unit];
- $Nx$ - number of lines (service resources, positions, capacity) of device x.

For the better understanding of the model and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [10]) is used. For example $dem.F$ for demand flow; $inc.Y$ stands for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic.

### 2.2 Types and names of the base virtual devices

The graphic representations of the base virtual devices together with their names and types are shown in Fig. 2 (see [19]). The type of each of the basic virtual devices is also shown in Fig. 2. Each basic virtual device belongs to one of the following types: Generator, Terminator, Modifier, Server, Enter Switch, Switch and Graphic connector. With the exception of the Switch, which has one or two entrances and one or two exits, every other virtual device has one entrance and/or one exit.

In the conceptual model each virtual device has a unique name. The names of the devices are constructed according to their position in the model. The model is partitioned into service stages (dialing, switching, ringing and communication). Every service stage has branches (enter, abandoned, blocked, interrupted, not available, carried), corresponding to the modeled possible cases of ends of the calls' service in the branch considered. Every branch has two exits (repeated, terminated) which show what happens with the calls after they leave

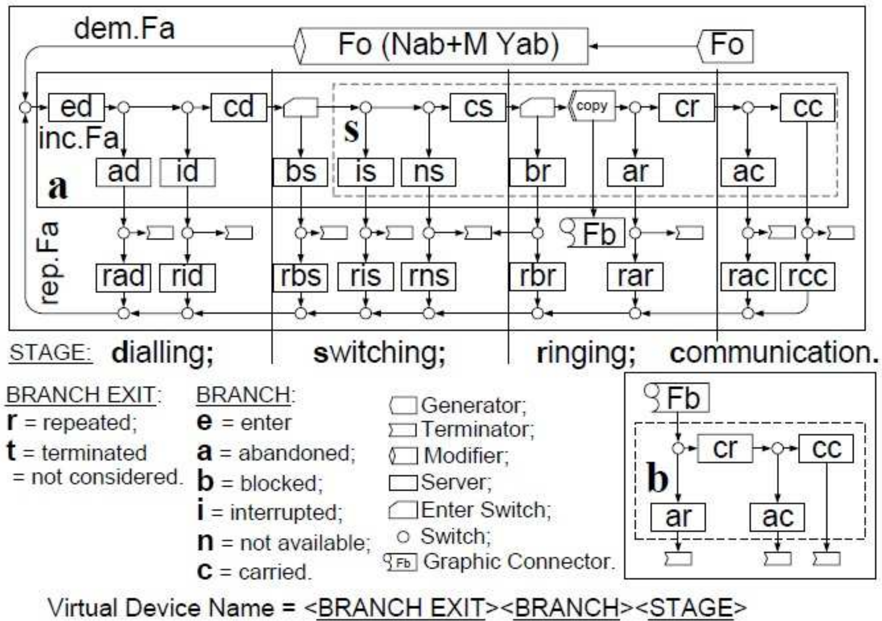


Fig. 2: Classical conceptual model of an overall telecommunication system (see [19]).

the telecommunication system. Users may make a new bid (repeated call), or to stop attempts (terminated call). In virtual device name construction, the corresponding bold first letters of the names of stages, branches end exits are used in the order shown below.

Virtual Device Name = <BRANCH EXIT><BRANCH><STAGE>

A parameter's name of one virtual device is a concatenation of parameters name letter and virtual device name. For example, "Yid" means "traffic intensity in interrupted dialing case"; "Fid" means "flow (calls) intensity in interrupted dialing case"; "Pid" means "probability for interrupted dialing"; Tid = "mean duration of the interrupted dialing"; "Frid" = "intensity of repeated flow calls, caused by (after) interrupted dialing".

### 2.3 Comprise virtual devices

The following comprising virtual devices denoted by **a, b, s** (see Fig. 2) and **ab** (not shown in Fig. 2) are considered in the model.

– **a** comprises all calling terminals (A-terminals) in the system. It is shown with continuous line box in Fig. 2;

- **b** comprises all called terminals (B-terminals) in the system. It is shown in box with dashed line in the down right corner in Fig. 2;
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Fig. 2;
- **s** virtual device corresponding to the switching system. It is shown with dashed line box into the a-device in Fig. 2.

The flow of calls (B-calls), with intensity Fb, occupying the B-terminals, is coming from the Copy device. This corresponds to the fact that at the beginning of the ringing a second (B) terminal in the system becomes busy. The second reason for this conceptual modelling trick is that the paths of the A and B-calls are different in the telecommunication system's environment, after releasing the terminals (compare environments of **a** and **b** – devices in Fig. 2). There are two virtual devices of type Enter Switch (see Fig. 2) – before Blocked Switching (**bs**) and Blocked Ringing (**br**) devices. These devices deflect calls if there is no free line in the switching system or the intent B-terminal is busy, respectively. The correspondent transition probabilities depend on the macrostate of the system ($Yab$). The macrostate of a (virtual) device (including the overall network, considered as a device) is defined as the mean number of simultaneously served calls in this device, in the observed time interval (similar to "mean traffic intensity"in (see [10]).

### 2.4 Paths of the Calls

The network under consideration corresponds to the reference configuration "terminal - subscriber switch - terminal" [14]. We ignore the signalling network. In the present paper, "call" means "call attempt" or "bid" according to [10]. The paths of the calls, generated from (and occupying) the A-terminals in the proposed network traffic model and its environment are shown in Fig.2. $Fo$ is the intent intensity of calls of one idle terminal; $M$ is a constant, characterizing the BPP (Bernoulli-Pascal-Poisson) flow of demand calls ($dem.Fa$). If $M = -1$, the intensity of demand flow corresponds to Bernoulli (Engset) distribution. If $M = 0$ – to the Poisson (Erlang) distributio. If $M = +1$ – to the Pascal (Negative Binomial) distribution. In our analytical model every value of $M$ in the interval $[-1, +1]$ is allowed. The BPP-traffic model is very applicable (see [14]) (the handbook is a clear comparison basis for the main ideas, discussed in this paper). In the numerical examples, presented in [19, 21], $M = 0$, because the conclusions made are independent of the input flow model.

### 2.5 Formulation of general teletraffic tasks

**System tuples and base tuples.** In the conceptual model presented we have at least 35 important virtual devices (31 base and 4 (**a, b, ab** and **s**) comprising). These devices are important because values of their parameters are specific for the characteristics and state of the modeled telecommunication system. Since every device has 5 parameters $(P, F, T, Y, N)$, the total number of parameters is 175.

**Definition 1.** *A system tuple is a finite set of distinguishable (by name and/or position) parameters' values, which fulfills simultaneously the three following requirements:*

1. *All parameters (parameters' set), evaluated by the system tuple, correspond to one considered (observed, modeled) system;*
2. *All the values of a system tuple correspondent to the one and the same time interval of measurements or considerations;*
3. *The instant of beginning and duration of this time interval are elements of the system tuple set.*

Every subset of a system tuple is called *subtuple*.

There are many obvious dependencies in a system tuple, corresponding to the Full Parameters' Set of the Conceptual Model. For example, the sum of probabilities of outgoing transitions in every virtual switch devices has value one; in stationary state Little's formula ($Y = FT$) is in force for every virtual device; we assume most of devices with infinite capacity. As a result, there are sets of base parameters (sub-tuples), with the following property: If we know the values of the base parameters, we may calculate the values of all other parameters of the same system tuple. Several different base parameters' sets may exist. After careful analysis and some assumptions (see below) we have chosen a base parameters' set with 41 parameters. The values of these parameters we call base tuple. The base tuple is a sub-tuple of a system tuple.

**Classification of the parameters.** The parameters of the chosen base parameters' set may be classified, according origination of their values, in the following fice groups:

1. Human Behaviour Parameters are 21: $Fo, Nab, Prad, Tid, Prid, Pris, Tis,$ $Pns, Tns, Prns, Tbs, Prbs, Tbr, Prbr, Par, Tar, Prar, Tcr, Prac, Tcc, Prcc$;
2. Technical Characteristics Parameters are 4: $Pid, Pis, Tcs, Ns$;
3. Mix Factors' Parameters are 6: $Ted, Pad, Tad, Tcd, Pac, Tac$;
4. Modeler Chosen Values Parameter (1): M;
5. Derived Parameters from the previous four groups are 9: $Yab, Fa, dem.Fa,$ $rep.Fa, Pbs, Pbr, ofr.Fs, Ts, ofr.Ys$.

In this paper, we propose a short term classification of the chosen base parameters' set with 31 static and 10 dynamic parameters. For the static parameters we assume that their values don't depend on the state of the system and correspondingly on the intensity of the input flow. They may depend on other factors, e.g. the time of the day; seasons, human temperament, Telecom Administration, Gross Domestic Product and so on, but for the observed and modeled time interval we consider them as constants. The 31 static parameters are: $M, Nab, Ns, Ted, Pad, Tad, Prad, Pid, Tid, Prid, Tcd, Tbs, Prbs, Pis, Tis, Pris,$ $Pns, Tns, Tcs, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc.$

The 10 dynamic parameters, with mutually dependent values are: $Fo$, $Yab$, $Fa$, $dem.Fa, rep.Fa, Pbs, Pbr, ofd.Fs, Ts, ofr.Ys.$

**Stationary teletraffic tasks.** For a short observation interval, we usually consider processes, in the investigated telecommunication system, as standing in a stationary state, described with a system tuple. Some of the values of the system tuple may be known (measured or stipulated), others – not known. The proposed parameters' classification allows definition of different teletraffic tasks' types. Depending on the task specificity, Mix Factors' Parameters may be considered as belonging to the Human Behaviour or Technical Characteristics Parameters groups. Since, excluding M, we have three groups of parameters' types and corresponding main stationary teletraffic tasks' types:

1. *System State Task* is to find values of the 5th group of parameters, if the values of the rest base parameters, from the same base tuple, are known. Note that $Yab$ is the macrostate of the system and the values of $Pbs$ and $Pbr$ belong to Quality of Service (QoS) parameters;
2. *Technical characteristics task* is to find values of the 4th group of parameters, if the values of the rest base parameters, from the same base tuple, are known. Note that $Pid$ and $Pis$ are caused from technical failures usually, $Tcs$ is limited and $Ns$ is a main network dimensioning parameter. The Network Dimensioning Task (NDT) is for finding $Ns$ if the target values of $Pbs$ and $Pbr$ are known;
3. *Human Behaviour Task* is to find values of the 1st group of parameters, if the values of the rest base parameters, from the same base tuple, are known. This task is difficult due the relatively big number of unknown values. There are some results for finding important parameters as the number of active terminals $Nab$ in [18].

Others criteria for parameters' classification and correspondent teletraffic tasks, inside a system tuple, are theoretically and practically interesting, too. For example Task for Inconvenient Measurements is to find values of the difficulty measured parameters, e. g. intensity of repeated attempts flow, if the values of easy measured parameters are known.

**Dynamic teletraffic task.** The system's dynamic may be presented with a series of system tuples. There is a difference between long and short term dynamics. In long term considerations, all the system parameters may be with variable values. In short term analysis, some of parameters may be considered as having constant values.

### 2.6 Main assumptions

Due to the complexity of the model, in order to obtain relatively simple analytical expressions about the parameters, after a careful analysis the following assumptions are formulated:

A-1. (Closed System Structure) We consider a closed telecommunication system with functional structure shown in Fig. 2;

A-2. (Device Capacity) All base virtual devices in the model have unlimited capacity. Comprising devices are limited: **ab**-device contains all the active terminals $Nab \in [2, \infty]$; switching system (**s**) has capacity of $Ns$ calls (every internal switching line may carry only one call); every terminal has capacity of one call, common for both incoming and outgoing calls;

A-3. (A-Terminal Occupation) Every call, from the flow incoming in the telecommunication system ($inc.Fa$), falls only on a free terminal. This terminal becomes a busy A-terminal;

A-4. (Stationarity) The system is in stationary state. This means that in every virtual device in the model (including comprising devices like switching system), the intensity of input flow $F(0, t)$, call holding time $T(0, t)$ and traffic intensity $Y(0, t)$ in the observed interval $(0, t)$ converge to the correspondent finite numbers $F, T$ and $Y$, when $t \to \infty$. In this case we may apply the theorem of Little (1961) and for every device: $Y = FT$;

A-5. (Calls' Capacity) Every call occupies one place in a base virtual device, independently from the other devices (e.g. a call may occupy one internal switching line, if it find free one, independently from the state of the intent B-terminal (busy or free));

A-6. (Environment) The calls in the communication systems' environment (outside the blocks a and b in Fig. 2) don't occupy any telecommunication systems' device and therefore they don't create communication systems' load. (For example, unsuccessful calls, waiting for the next attempt, are in "the head" of the user only. The calls and devices in the environment form the intent and repeated calls flows). Calls leave the environment (and the model) in the instance they enter a Terminator virtual device;

A-7. (Parameters' Independability) We consider probabilities for direction of calls to, and holding times in the base virtual devices as independent of each other and from intensity $Fa = inc.Fa$ of incoming flow of calls. Values of these parameters are determined by users' behavior and technical characteristics of the communication system. (Obviously, this is not applicable to the devices of type Enter Switch, correspondingly to $Pbs$ and $Pbr$.);

A-8. (Randomness) All variables in the analytical model may be random and we are working with their mean values, following the Theorem of Little.

A-9. (B-Terminal Occupation) Probabilities of direction of calls to, and duration of occupation of devices **ar, cr, ac** and **cc** are the same for A and B-calls;

A-10. (Channel Switching) Every call occupies simultaneously places in all the base virtual devices in the telecommunication system (comprised of devices **a** or **b**) it passed through, including the base device where it is in the moment of observation. Every call releases all its occupied places in all base virtual devices of the communication system, in the instant it leaves comprising devices **a** or **b**.

A-11. (Homogeneity of the terminals) All terminals are homogeneous, i.e., for every terminal all corresponding characteristics are equal.

A-12. (Direction of the A-calls) Every A-terminal generates all call attempts only towards other terminals, not towards itself.

A-13. (Ordinadirity of the B-flow) The flow directed to the B-terminals ($Fb$) is ordinary. The only exception is when two or more calls reach a free B-terminal simultaneously.

A-14. (Probability of repeated calls blocking) The probability $Pbr$ for finding the B-terminal busy is one and the same for the first and all of the next repeated attempts.

## 3 QoS prediction task

### 3.1 QoS prediction task formulation

We consider the conceptual model presented in Fig. 2 and described in the previous section. In this chapter, we consider that the overall telecommunication system provides four services: (1) finding B-terminal; (2) connection to B-terminal; (3) finding B-user (with sound, vibration, message, etc.); and (4) transmission and/or record of messages. The quality of these services depends on many subsystems including the users' behaviour and network.

**Types of Parameters.** There are two types of parameters – static and dynamic. The 10 basic dynamic parameters (with values dependent on the system state) are: $Fo$, $Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fs, Ts$, and $ofr.Ys$. All other dynamic parameters can be obtained from these.

Note that the traffic $Yab$ of all terminals is accepted as a system macro-state parameter.

**Input Parameters.** These are mostly static, i.e., related to the network technical characteristics or the users' behaviour. We choose one dynamic parameter - $Fo$ (the intent intensity of calls of one idle terminal) as an independent input variable. The proposed analytical model allows to find all dynamic values, if Fo and all static parameters are known.

The probability of finding the B-user is considered static (i.e. independent of the system state).

**Basic QoS parameters.** The basic QoS output parameters are:

- Quality of finding the B-terminal service, represented by the probability of call blocking due to lack of resources (equivalent network switching lines)– blocked switching ($Pbs$);
- Quality of connection to the B-terminal, represented by the probability of call blocking due to busy B-terminal – blocked ringing ($Pbr$).
- Network call efficiency ($Ec$);
- Network time efficiency ($Et$);
- Network traffic efficiency ($Ey$).

These two parameters allow determination of many other QoS indicators, related to traffic, time, and flow characteristics of users and terminals.

The goal of this section is to find analytically all unknown basic dynamic parameters, including the basic QoS output parameters.

### 3.2 Derivation of equations about the dynamic parameters

Here, we shortly present the analytical model of overall telecommunication system from [19]. All other parameters in the equations, except the dynamic parameters, are considered known.

**Theorem 1.** *The traffic intensity of all the terminals ($Yab$) is a sum of the traffic intensities of the A ($Ya$) and B-terminals ($Yb$):*

$$Yab = Ya + Yb\,. \tag{1}$$

Proof: i) There are no other terminals apart from the included in Fig. 1, because the modeled system is closed (Assumptions A-1); ii) Every terminal, at a given time moment, may be free or busy. If it is busy, it may be calling (A) or called (B), but not simultaneously both calling and called, because every terminal has capacity of one call (A-2). Obviously, from i) and ii) follows (1). □

Since the number of the terminals is limited to $Nab$ (assumption A-2), and there is no negative occupancy, we have the following obvious terminal traffic limitations in the studied system:

$$0 \leq Yab \leq Nab\,. \tag{2}$$

Inequations 2 are referred to as *absolute terminal traffic limitation*

**Theorem 2.** *The calls flow intensity occupying all terminals ($Fab$) is a sum of the intensities of calls flows occupying the A-terminals ($Fa$) and calls flow occupying B-terminals ($Fb$):*

$$Fa + Fb = Fab\,. \tag{3}$$

Proof: This follows from assumption A-1 and the conceptual model, in which $Fb = inc.Fb$ is a flow different from $Fa$ and corresponds to the real cases when successful calls, after dialing and switching, occupy the B-terminals. From assumption A-3 follows that $Fa = inc.Fa$ (there are no blocked calls, directed to the communication system). □

**Theorem 3.** *The traffic intensity of the B-terminals ($Yb$) can be determined from the equation*

$$Yb = Fb\,Tb\,, \tag{4}$$

*where $Tb$ is the mean holding time of calls in a B-terminal, and:*

$$Fb = Fa(1 - Pad)(1 - Pid)(1 - Pbs)(1 - Pis)(1 - Pns)(1 - Pbr)\,, \tag{5}$$

$$Tb = Par\,Tar + (1 - Par)[Tcr + Pac\,Tac + (1 - Pac)Tcc]\,. \tag{6}$$

Proof: Equation (4) is Little's formula for device b in stationary state (A-4). Equation (5) expresses the fact that the A-calls have to avoid the six modeled losses before occupying the intent B-terminals, with mean intensity of calls $Fb$. Equation (6) is direct corollary from Fig. 2 (box for b-device, in the right-down), closed system structure (A-1), calls' capacity (A-5), excluding calls in the environment (A-6) parameters independability (A-7), randomness (A-8) and B-terminal occupation assumption (A-9).

We may derive the expression 6 for the B-terminals holding time ($Tb$) from the next considerations. From Fig. 2, parameters independability (A-7) and channel switching (A-10), it follows that $Yb$ is a sum of the traffics of the base blocks, comprised in it. The assumption for B-terminal occupation (A-9), implies that $Yar, Ycr, Yac$ and $Ycc$ are the same traffic intensities for A and B-terminals, so:

$$Yb = Yar + Ycr + Yac + Ycc. \tag{7}$$

On the other hand, we may express the traffic intensities using the Little's formula and presenting every flow intensity in the base devices as a function of $Fb$:

$$Yar = Far\,Tar = Fb\,Par\,Tar\,; \tag{8}$$

$$Ycr = Fcr\,Tcr = Fb(1 - Par)Tcr\,; \tag{9}$$

$$Yac = Fac\,Tac = Fb(1 - Par)Pac\,Tac\,; \tag{10}$$

$$Ycc = Fcc\,Tcc = Fb(1 - Par)(1 - Pac)Tcc\,. \tag{11}$$

After replacing (8), (9), (10) and (11) in (7), taking in consideration A-9 and using (4) we obtain (6). □

**Theorem 4.** *The A-terminals' traffic intensity ($Ya$) is can be determined using the Little's formula*

$$Ya = Fa\,Ta\,, \tag{12}$$

*where $Ta$ is given by:*

$$\begin{aligned} Ta = Ted + Pad\,Tad + (1 - Pad)(Pid\,Tid + (1 - Pid) \\ \cdot(Tcd + Pbs\,Tbs + (1 - Pbs)(Pis\,Tis + (1 - Pis) \\ \cdot(Pns\,Tns + (1 - Pns)(Tcs + Pbr\,Tbr + (1 - Pbr)Tb))))). \end{aligned} \tag{13}$$

The proof of the theorem is very similar to the proof of Theorem 3, but includes more base devices, shown on Fig. 2.

**Theorem 5.** *Distinguishing static and dynamic parameters, we have:*

$$Ya = Fa[Sa_1 - Sa_2(1 - Pbs)Pbr - Sa_3 Pbs]\,, \tag{14}$$

*where*

$$Sa_1 = Ted + Pad\,Tad + (1 - Pad)[Pid\,Tid + (1 - Pid)[Tcd + Pis\,Tis +$$

$$+(1-Pis)[Pns\,Tns+(1-Pns)[Tcs+Tb]]]]\,,$$

$$Sa_2=(1-Pad)(1-Pid)(1-Pis)(1-Pns)[Tb-Tbr]\,,$$

$$Sa_3=(1-Pad)(1-Pid)[Pis\,Tis+(1-Pis)[PnsTns+(1-Pns)$$

$$\cdot[Tcs+Tb]]]-(1-Pad)(1-Pid)Tbs\,. \qquad (15)$$

Proof: Equations (14) and (15) are result of simple mathematical transformations of (13), after application of the Little's formula $Ya = FaTa$. □

**Theorem 6.** *The traffic of all simultaneously busy terminals ($Yab$) may be determined from equation 16 as a function of $Fa$ and other parameters:*

$$Yab=Fa\{Ted+Pad\,Tad+(1-Pad)[Pid\,Tid+(1-Pid)[Tcd+Pbs\,Tbs$$

$$+(1-Pbs)[Pis\,Tis+(1-Pis)[Pns\,Tns+(1-Pns)[Tcs+Pbr\,Tbr+2(1-Pbr)Tb]]]]]\}\,, \qquad (16)$$

*or equation (17), after separtion of static from dynamic parameters in it:*

$$Yab=Fa[S_1-S_2(1-Pbs)Pbr-S_3Pbs]\,, \qquad (17)$$

*where:*

$$S_1=Ted+Pad\,Tad+(1-Pad)[Pid\,Tid+(1-Pid)[Tcd+PisTis$$

$$+(1-Pis)[Pns\,Tns+1-Pns[Tcs+2Tb]]]]\,,$$

$$S_2=(1-Pad)(1-Pid)(1-Pis)(1-Pns)[2Tb-Tbr]\,,$$

$$S_3=(1-Pad)(1-Pid)[Pis\,Tis+(1-Pis)[Pns\,Tns+(1-Pns)[Tcs+2Tb]]]$$

$$-(1-Pad)(1-Pid)Tbs\,. \qquad (18)$$

Proof: Adding equations (4) and (13), using (1), and after elementary mathematical transformations, we obtain (16) and from it (17) and (18). □

We have to determinate the mean intensity of the input flow to the telecommunication system. This is the flow occupying the calling (A) terminals $Fa$. From the ITU E.600 definitions and Fig. 2 it is obvious that the intensity of incoming flow is a sum of the intensities of primary (demand) calls ($dem.Fa$) and repeated attempts ($rep.Fa$):

$$Fa=dem.Fa+rep.Fa\,. \qquad (19)$$

From the definition of BBP-flow we have (see Section 2.4):

$$dem.Fa=Fo(Nab+M\,Yab)\,. \qquad (20)$$

The proof of the following theorem can be found in [21, 19].

**Theorem 7.** *The intensity of the flow of repeated call attempts rep.Fa may be determined from the expression (21):*

$$rep.Fa = Fa\{Pad\,Prad + (1-Pad)[Pid\,Prid + (1-Pid)[Pbs\,Prbs + (1-Pbs) \cdot[Pis\,Pris + (1-Pis)[Pns\,Prns + (1-Pns)[Pbr\,Prbr + (1-Pbr) \cdot[Par\,Prar + (1-Par)[Pac\,Prac + (1-Pac)Prcc]]]]]]]\}\,. \quad (21)$$

After separation of static and dynamic parameters in the expression for $rep.Fa$ above, we obtain the following representation of $rep.Fa$ as a function of the dynamic parameters $Fa$, $Pbr$ and $Pbs$:

**Theorem 8.**

$$rep.Fa = Fa[R_1 - R_2 Pbr(1-Pbs) - R_3 Pbs]\,, \quad (22)$$

where

$$Q = Par\,Prar + (1-Par)[Pac\,Prac + (1-Pac)Prcc]\,,$$

$$R_1 = Pad\,Prad + (1-Pad)(Pid\,Prid + (1-Pid)Pis\,Pris + (1-Pis)(Pns\,Prns + (1-Pns)Q)\,,$$

$$R_2 = (1-Pad)(1-Pid)(1-Pis)(1-Pns)(Prbr-Q)\,,$$

$$R_3 = (1-Pad)(1-Pid)[PisPris + (1-Pis)[PnsPrns + (1-Pns)Q] - Prbs]\,. \quad (23)$$

In the teletraffic engineering of all the telecommunication networks, parameters characterizing the terminal traffic are used. One of the most important of them is the probability of finding the called terminal (B-terminal) busy ($Pbr$). The following theorem, proved in [19], gives analytical expression for this probability:

**Theorem 9.** *The probability of finding the B-terminal busy (Pbr) is*

$$Pbr = \begin{cases} \frac{Yab-1}{Nab-1}\,, & \text{if } 1 < Yab \le Nab\,. \\ 0\,, & \text{if } 0 \le Yab \le 1\,. \end{cases} \quad (24)$$

Using the conceptual model and the assumptions made, in [19] the unknown value of the mean blocking probability ($Pbs$), due to insufficient equivalent switching lines ($Ns$) is expressed analytically.

**Theorem 10.** *The mean holding time of the switching system (Ts) is given by the equation*

$$Ts = [Pis\,Tis + (1-Pis)[Pns\,Tns + (1-Pns)[Tcs + Pbr\,Tbr + (1-Pbr)Tb]]] = S_{1,z} - S_{2,z} Pbr\,, \quad (25)$$

*where*

$$S_{1,z} = Pis\,Tis + (1 - Pis)[Pns\,Tns + (1 - Pns)(Tb + Tcs)]$$
$$S_{2,z} = (1 - Pis)(1 - Pns)(Tb - Tbr)\,. \tag{26}$$

**Theorem 11.** *The intensity of the offered flow of call attempts ($ofr.Fs$), may be expressed through equation (27):*

$$ofr.Fs = Fa(1 - Pad)(1 - Pid)\,. \tag{27}$$

**Theorem 12.** *The probability of blockedg switching (Pbs) is determined from equations (28) and (29):*

$$ofr.Ys = ofr.Fs\,Ts\,, \tag{28}$$

$$Pbs = Erl_b(Ns, ofr.Ys)\,. \tag{29}$$

Equation (29) simply expresses the use of the Erlang-B formula for determination of the blocking probability in the switching system, on the basis of the number of internal switching lines ($Ns$) and offered traffic $ofr.Ys$. The expression $Erl_b(Ns, ofr.Ys)$ denotes the famous formula of Erlang:

$$Erl_b(Ns, ofr.Ys) = \frac{(ofr.Ys)^{Ns}}{Ns!} / \sum_{j=0}^{Ns} \frac{(ofr.Ys)^j}{j!}\,. \tag{30}$$

The following theorem can be easily verified using the graphic representation of the conceptual model.

**Theorem 13.** *The intensity of the carried call attempts by the switching system satisfies the equation:*

$$crr.Ys = (1 - Pbs)ofr.Ys\,. \tag{31}$$

### 3.3 A system of equations characterizing the overall state of the telecommunication system

For the 11 basic dynamic parameters, which are mutually dependent: $Fo$, $Yab$, $Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fs, Ts, ofr.Ys$, in the previous subsection we have derived 10 equations with 6 generalized static parameters. These are: equation (17) for $Yab$; (19) for $Fa$; (20) for $dem.Fa$; (22) for $rep.Fa$; (24) for $Pbr$; (25) for $Ts$; (27) for $ofr.Fs$; (28) for $ofr.Ys$; (29) for $Pbs$ and (31) for $crr.Ys$. We have no equation in which $Fo$ is present in the left-hand side. It is in the right-hand side in (20) only. The system becomes:

$$Yab = Fa[S_1 - S_2(1 - Pbs)Pbr - S_3 Pbs]\,, \tag{32}$$

$$Fa = dem.Fa + rep.Fa\,. \tag{33}$$

$$dem.Fa = Fo(Nab + M\,Yab)\,, \tag{34}$$

$$rep.Fa = Fa[R_1 - R_2 Pbr(1 - Pbs) - R_3 Pbs]\,, \tag{35}$$

$$Pbr = \begin{cases} \frac{Yab-1}{Nab-1}\,, & \text{if } \; 1 < Yab \leq Nab\,. \\ 0\,, & \text{if } \; 0 \leq Yab \leq 1\,, \end{cases} \tag{36}$$

$$Ts = S_{1,z} - S_{2,z} Pbr\,, \tag{37}$$

$$ofr.Fs = Fa(1 - Pad)(1 - Pid)\,, \tag{38}$$

$$ofr.Ys = ofr.Fs\,Ts\,, \tag{39}$$

$$Pbs = Erl_b(Ns, ofr.Ys)\,, \tag{40}$$

$$crr.Ys = (1 - Pbs)ofr.Ys\,, \tag{41}$$

The equations for the static parameters are:

$$\begin{aligned} S_1 = Ted + Pad\,Tad + (1 - Pad)[Pid\,Tid + (1 - Pid)[Tcd + Pis\,Tis \\ +(1 - Pis)[Pns\,Tns + 1 - Pns[Tcs + 2Tb]]]]\,, \end{aligned} \tag{42}$$

$$S_2 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)[2Tb - Tbr]\,, \tag{43}$$

$$S_3 = (1-Pad)(1-Pid)[Pis\,Tis + (1-Pis)[Pns\,Tns + (1-Pns)[Tcs + 2Tb]]]\,, \tag{44}$$

$$S_{1,z} = Pis\,Tis + (1 - Pis)[Pns\,Tns + (1 - Pns)(Tb + Tcs)]\,, \tag{45}$$

$$S_{2,z} = (1 - Pis)(1 - Pns)(Tb - Tbr)\,, \tag{46}$$

$$\begin{aligned} R_1 = Pad\,Prad + (1-Pad)(Pid\,Prid + (1-Pid)Pis\,Pris + (1-Pis)(Pns\,Prns \\ +(1 - Pns)Q)\,, \end{aligned} \tag{47}$$

$$R_2 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)(Prbr - Q)\,, \tag{48}$$

$$\begin{aligned} R_3 = (1 - Pad)(1 - Pid)[Pis\,Pris + (1 - Pis)[Pns\,Prns \\ +(1 - Pns)Q] - Prbs]\,, \end{aligned} \tag{49}$$

$$Q = Par\,Prar + (1 - Par)[Pac\,Prac + (1 - Pac)Prcc]\,. \tag{50}$$

### 3.4 Numerical results

Numerical results related to the QoS prediction task are shown in Fig. 3. The change of the network call efficiency ($Ec$), the traffic efficiency ($Ycc$), the traffic of the B-terminals ($Yb$), the traffic of the A-terminals ($Ya$), traffic effectiveness ($Ey$) and the probability of blocked ringing ($Pbr$) depending on the average traffic of one terminal in the system ($Yab/Nab$). The traffic values are shown as a ratio to the number of all terminals ($Nab$). The values belong to the whole theoretically possible interval for telecommunication systems without losses due to lack of resources. The values used for the input parameters (the static parameters) are typical ones for telephone systems.

The network call efficiency ($Ec$) is the ratio of the number of successive call attempts to the number of all call attempts.

The network traffic effectiveness ($Ey$) is the absolute of the carried traffic which is the useful target effect of the work of the telecommunication network. This is the traffic corresponding to the base virtual device **cc** (carried communication – successful call attempt). Obviously, it is a part of the traffic of the B-terminals ($Yb$) which includes other unavoidable load such as the signalisation.These two pieces of traffic have maximums at one and the same point of the system's load – 80.35% of the maximum. The values of all variables illustrated in Fig. 3 are shown for this particular point.

The traffic efficiency ($Ey$) is the ratio of the successfull traffic to whole traffic of the A-terminals, i.e. $Ey = Ycc/Ya$.

It can be seen that the traffic and call efficiencies have different values, including different maximums – 80.34% and approximately 62%, respectively. At the point of maximum theoretical value of the load (100%), there is no useful load and all efficiencies are equal to zero.

An important result is finding the maximum of the B-terminals' traffic ($Yb$) in networks without technical losses. This means that the required number of switching lines for guaranteeing of connection without blocking, is less than the theoretical – 50% of the number of the terminals ($Nab$). In our case, the required number of switching lines is 31% of $Nab$ (the maximum in Fig. 3 is 30.90%).

## 4 Dimensioning and redimensioning tasks

### 4.1 Network dimensioning task

One of the main problems which often have to solved by the operators of telecommunication services is to determine the volume of the telecommunication resources in such a way that it is sufficient for the servicing of a given input flow of requests with in advance specified characteristics of the Qualisty of Service (QoS) in the Service Level Agreement (SLA) [13]. The aim is for the user to be satisfied with the offered service at the arranged price [16, 17].

The dimensioning of a telecommunication network is used to determine the capacity of the nodes and switching lines in the network while looking for the balance between the requirements for reaching and sustaining a certain level of QoS and the final value of the offered service.

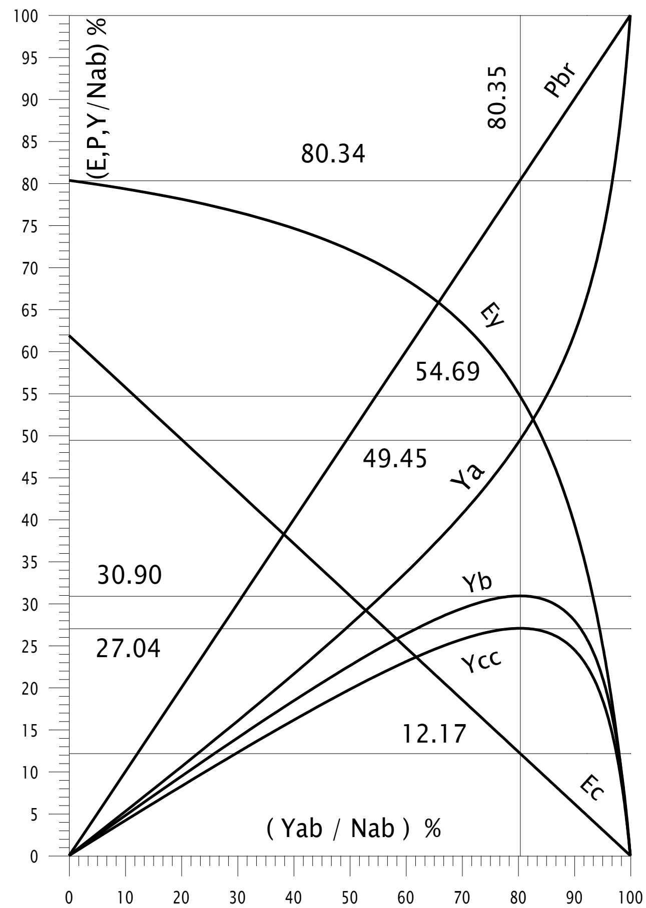


Fig. 3: Boundary and extremal values of the terminal traffic and the efficiency of overall telecommunication network with virtual channel switching without losses due to lack of resources .

**Formulation of the network dimensioning task – goals, problems, parameters.** By dimensioning of a network we mean the process of providing the necessary number of internal switching lines of the switching system ($Ns$), which are enough to provide the agreed in advance grade of the QoS.

In the process of determining the designed values of the number of internal switching lines ($dsn.Ns$), the probability of blocked switching ($Pbs$) is chosen as a target parameter. This parameter represents an important criterion for the Grade of Service (GoS). In a well-dimensioned net, the grade of the blocking due to the lack of resources, should not exceed a certain pre-aranged level of the QoS, i.e., the target value of the blocking ($trg.Pbs$). The target value ($trg.Pbs$) of the probability of blocking to the lack of resources is administratively prearanged in the Service Level Agreement (SLA) [15].

The problems for network dimensioning with respect to the length of the time interval of the envisaged planning (see [5–7, 11]) can be classified as tasks for:

– *Capacity management* [6], including prediction of the needed capacity, daily or weekly observation of the output of the network and short-term network adjustment.
– *Traffic management*, including dimensioning in short-term plan (up to 2 years). It mainly concerns re-configuration of already existing resources in the network.
– *Redimensioning* of network in which the dimensioning of the network concerns medium-term planning (up to 5 years).
– *Dimensioning* of networks with long-term planning or initial dimensioning of netowkrs (up to 10 years).

In all of the above dimensioning problems, for the purpose of the operative management and planning, the projected resources of the network are determined on the basis of determined, measured, prognostic and target values. This is the purpose of the method for dimensioning developed by us [31]. In our research, two of the total three types of dimensioning problems are studied. These are the Network Dimensioning Task (NDT) in long-term planning and the task for dimensioning of a network in medium-term planning, for repeated dimensioning, referred to as Network Redimensioning Task (NRDT).

The obtained results are applicable to all of the meantioned activities related to the dimensioning of networks. In the long-term dimensioning, methods which allow the prognosis of traffic parameters of the telecommunication system are used. The presented results are used in both of the problems [21].

**Classification and notation of the used parameters depending on the way of determining of their values.** In many of the problems of Teletraffic Engineering (TE) more than one different values of a certain parameter have to be used [36, 37]. The parameters in our research of system tuples and the sub-collection of the selected base parameters (base tuple) are classified according to way in which their values are determined. The proposed classification is presented in Table 1. The type of the denoted value is denoted by prefix – qualifier before the name of the parameter [18]. The models of the NDT and the NRDT have values of input parameters which are: empirical (obtained through measurement in real functioning networks) [33]; by assumption about the environment of the designed network; administratively determined; designed (for instance, characterizing the QoS). Their output parameters are the designed ($dsn.Pbr, dsn.ofr.Ys, dsn.Pbs, dsn.Ns$).

**Base assumptions in the mathematical modeling of the network dimensioning task.** On the basis of assumption A-8 and the following obesrvations, we work with the mean values of the teletraffic parameters.

1. The Little's formula, which is the basic formula used in the analytical modeling, does not depend on the distribution of the quantities in the model. This allows us to work with the mean values of the parameters.
2. The formula for determining of the probability of blocking of Poisson, Engset, the B and C formula of Erlang, etc., use only mean values of the input quanities, taking into account their assumed distributions.
3. The number of the designed internal switching lines is searched for as an integer number.

   A confirmation of the applicability of this approach are ITU recommendations [8–10, 12], etc, and, also, the Cisco instructions for traffic analysis and dimensioning of networks for VoIP [3].

We need to add the following additional assumptions to the 14 stated in the previous section [21]:

A-15. The probabilities $Pad, Pid, Pbs, Pis, Pns, Pbr, Par, Pac$ preserve their values during the repeated call attempts.

A-16. The probabilites of entering of repeated call attempts due to abandoned dialing $Prad$, interrupted dialing $Prid$, interrupted dialing $Prid$, interrupted switching $Pis$, not available switching $Prns$, abandoned ringing $Prar$ or abandoned communication $Prac$, which characterize the users' behavior also preserve their values.

Parameters characterizing the users' behavior are the mean service time of the separate devices: $Ted$, $Tad$, $Tid$, $Tcd$, $Tbs$, $Tis$, $Tns$, $Tcs$, $Tbr$, $Tar$, $Tcr$, $Tac$ and $Tcc$.

A-17. The mean service time in the various devices, i.e., $Ted$, $Tad$, $Tid$, $Tcd$, $Tbs$, $Tis$, $Tns$, $Tcs$, $Tbr$, $Tar$, $Tcr$, $Tac$ and $Tcc$, remain the same.

As a result of the assumptions A-6 and from A-15 to A-17, the 8 generalized static parameters $S_1$, $S_2$, $S_3$, $R_1$, $R_2$, $R_3$, $S_{1,z}$ and $S_{2,z}$ do not change their values before and after the change of the number of the switching lines.

| **Name according to the origin of the values** | **Used prefix - qualifier** | **Example** |
|---|---|---|
| empirical values, (primary parameter, secondary parameter, derived) | *emp.* | *emp.crr.Ys - empirical values of carried traffic in the system* |
| Designed values | | |
| based on assumption | *ass.* | *ass.Fo – values of input flow, generated by one idle terminal, determined by assumption* |
| target | *trg.* | *trg.Pbs – target values of probability blocked switching due to lack of resources* |
| threshold | *thr.* | *thr.Fo – threshold values of input flow, generated by one idle terminal* |
| administrative | *adm.* | *adm.Nab – administratively determined number of terminals* |
| designed | *dsn.* | *designed (project) values of:*<br>- *offered traffic: dsn.ofr.Ys;*<br>- *probability of busy terminals: dsn.Pbr;*<br>- *required number of switching lines in the telecommunication system: dsn.Ns.* |
| Test values | | |
| test | *tst.* | *tst.Pbs – test received values* |

Table 1: Notation of the parameters' values according to their origin.

**NDT/NRDT problem statement.** The solution of any of the two tasks – NDT and NRDT – requires the achievement of the following goal:

– To determine the required number of switching lines ($dsn.Ns$) in a working network so that an agreed in advance with the users QoS ($trg.Pbs$) is achieved or maintained.

The following subproblems have to be solved in the network dimensioning task:

1. Finding of the designed values of parameters ($dsn.Pbr, dsn.ofr.Ys$) through the least possible number of empiric values of easily measurable variables ($crr.Ys, emp.Fo$) which describe the designed state of the network (and represent a key for the evaluation of the designed number of internal switching lines, for example $dsn.Yab, dsn.ofr.Ys$) [20].
2. Finding the designed number of switching lines ($dsn.Ns$), required for the service of the projected offered traffic, on the basis of the evaluated designed values of the parameters. This should be done in such a way that the designed values for blocking due to lack of resources ($dsn.Pbs$) does not exceed the target value $trg.Pbs$, i.e. $dsn.Pbs \leq trg.Pbs$ where $dsn.Pbs = Erl_b(dsn.Ns, dsn.ofr.Ys)$.

Various formula exist for the evaluation of the number of internal switching lines in teletraffic theory. We use the Erlang's B-formula recommended by ITU (see Theorem 12, equation 30). It represents a functional dependence between the number of switching lines, the offered traffic and the grade of service (the probability of blocking as a measure of quality of traffic service).

The users' behavior in the model used for solving the dimensioning task is described by the following parameters: intent flow intensity of one idle terminal $Fo$, the probabilities for losses, the probabilities of entering of repeated call attempts due to service refusal and the mean duration of occupying of the separate devices by the requests. Other variables that characterize the users' behavior are the probabilities of losses in the different stages: losses due to abandoned dialing $Pad$, interrupted dialing by the system $Pid$, losses due to interrupted switching $Pis$ or not available switching $Pns$, due to abandoned ringing $Par$, probability of abandoned communication $Pac$, i.e., all probabilities for losses with the exception of the probability for blocking due to lack of resource $Pbs$ and the probability of blocked ringing $Pbr$.

**Solving the network dimensioning/redimensioning task.** In [21], some dependencies are derived and proven which allow using the least number easily measurable dynamic parameters ($emp.crr.Ys, emp.Pbs$), values of administratively determined parameters: ($adm.Nab$ and $M$ – modifier for the BPP input flow) and the eight generalized static parameters ($S_1, S_2, S_3, R_1, R_2, R_3, S_{1,z}, S_{2,z}$) to evaluate the empirical values of the intensity of the input flow, generated by one idle terminal $Fo$ (Theorem 14) which is needed to determine the designed values in the NDT/NRDT.

**Theorem 14.** *The empirical value of the intensity of the flow generated by one idle terminal emp.Fo given $Pbr \neq 0$ can be determined by*

$$Fo = \frac{crr.Ys[(1 - R_1 - R_3\,Pbs)S_{2,z} - R_2\varOmega]}{(1 - Pad)(1 - Pid)Ts[(1 - Pbs)S_{2,z}(Nab + M) + M(Nab - 1)\varOmega]}\,, \quad (51)$$

*where $\varOmega = (1 - Pbs)S_{2,z}\,Pbr$.*

*When $Pbr = 0$, $Fo$ is given by*

$$Fo = \frac{crr.Ys[1 - R_1 - R_3\,Pbs]}{(1 - Pad)(1 - Pid)(1 - Pbs)S_{1,z}Nab + M(S_1 - S_3\,Pbs)crr.Ys}\,, \quad (52)$$

*given that $Pbs = emp.Pbs$ and $crr.Ys = emp.crr.Ys$.*

*Note:* When there are occupied terminals in the system but there are no losses due to occupied terminal, i.e. if $Pbr = 0$ and respectively $0 \leq Yab \leq 1$, then $0 < Fo \leq thr.Fo$. In the case of losses due to finding the B-terminals busy, we have $thr.Fo < Fo$.

**Computation of designed parametric values in the NDT/NRDT.** In [21] we have proposed analytical method and algorithm for dimensioning (finding the number of switching lines) of overall telecommunication network.

*Note*: We consider that the following parameters have constant values:

1. The number of active terminals ($adm.emp.Nab = adm.dsn.Nab$) which is denoted by $Nab$.
2. The designed and empiric values of the activity of one terminal $dsn.Fo = emp.Fo$ which is denoted by $Fo$.
3. The values of $S_1, S_2, S_3, R_1, R_2, R_3, S_{1z}$ and $S_{2,z}$ are considered constant because they are relatively independent on the system state in the time interval considered.

For brevity, in the derivation of the analytical expressions and when referring to them we will use $Pbs$ instead of $trg.Pbs$.

**Theorem 15.** *Given BPP type of input flow, in the ND/NRD task the designed value of the probability of blocked ringing (dsn.Pbr), when $thr.Fo < Fo$, satisfies the system:*

$$\left| \begin{array}{l} A(dsn.Pbr)^2 + B\,dsn.Pbr + C = 0 \\ dsn.Pbr\ \in\ [0;1] \end{array} \right. , \quad (53)$$

*where*

$$A = (dsn.Fo\,M S_2 - R_2)(1 - Pbs)(Nab - 1)\,,$$

$$B = (1 - Pbs)[dsn.Fo\,S_2(Nab + M) - R_2] + (Nab - 1)[(1 - R_1 - R_3Pbs) - dsn.Fo\,M(S_1 - S_3)]\,,$$

$$C = (1 - R_1 - R_3Pbs) - dsn.Fo(Nab + M)(S_1 - S_3Pbs).$$

In [34] on the basis of the assumptions in the NDT/NRDT and the corresponding notation, it is shown that the values of $dsn.Pbr$ can be evaluated and they depend on the measured values of the parameters $dsn.Fo$, $Nab$ and $trg.Pbs$. When $0 < trg.Fo \leq thr.Fo$ then $0 \leq Yab \leq 1$ and based on equation 24 from Theorem 9, it follows that $Pbr = 0$ [34].

In [21] it is proved that the equation in (15) has only one real root which satisfies the condition $Pbr \in [0, 1]$.

The following theorem is proven in [32, 27]:

**Theorem 16.** *The value of $dsn.ofr.Ys$ can be determined under the assumptions and the conditions of the NDT/NRDT using equations the following equations:*

$$dsn.ofr.Ys = \frac{Fo(1 - Pbr)(1 - Pad)(1 - Pid)(S_{1,z} - S_{2,z} Pbr)}{\begin{array}{r} Fo(1 + M\,Pbr)[S_1 - S_2(1 - Pbs)Pbr - S_3\,Pbs] \\ -\,Pbr[1 - R_1 - R_2(1 - Pbs)Pbr - R_3\,Pbs] \end{array}}\,, \tag{54}$$

*when $thr.Fo < Fo$ and*

$$dsn.ofr.Ys = \frac{Fo\,Nab(1 - Pad)(1 - Pid)S_{1,z}}{1 - R_1 - R_2 - R_3\,Pbs - Fo\,M(S_1 - S_3\,Pbs)}\,, \tag{55}$$

*when $0 < Fo \leq thr.Fo$.*

Derived and proven in [21] are also formula for evaluation of the following designed traffic parameters: $dsn.Fa, dsn.dem.Fa, dsn.rep.Fa, dsn.Yb, dsn.Ya, dsn.Tb, dsn.ofr.Fs, dsn.crr.Ys$ etc. Also, their change in the whole theoretical interval is shown graphically.

We will shortly summarize the method for evaluation of the required number of switching lines ($dsn.Ns$) in NDT/NRDT. Based on a method proposed by us, from the expression for $dsn.ofr.Ys$ (Theorem 16) using the inverse B-formula of Erlang ($inv.Erl_b$), the required number of switching lines given the conditions of the NDT/NRDT can be determined directly [21]:

$$dsn.Ns = inv.Erl_b(dsn.ofr.Ys, trg.Pbs)\,. \tag{56}$$

All of the derived equations are valid for the whole theoretically allowed interval of the parameters. The problems for existence and uniqueness of the solutions are are also studied in [21]. The necessary and sufficient conditions for the evaluation of $dsn.Ns$ based on the B-formula of Erlang are derived.

**Comparison of the proposed method with some widely accepted and used methods for dimensioning/redimensioning.** Studied and quantitatively compared in [26] are some of the widely accepted and used methods for dimensioning/redimensioning of networks which are based on two recommendations of ITU [9, 10] for offered traffic and those of CISCO [3]. The results obtained using the method proposed by us are compared with them in [35].

*Main goal of the method for comparison:* To determine the test values of the blocking probabilities due to lack of resources ($test.Pbs$) in the different methods for dimensioning/redimensioning with the desired QoS ($trg.Pbs$).

One and the same methodology is used for all methods, which is based on the equality:

$$test.Pbs(dsn.Ns, test.ofr.Ys) = Erl_b(dsn.Ns, test.ofr.Ys)\,, \qquad (57)$$

where $test.ofr.Ys$ is a test offered traffic for the telecommunication system considered and $dsn.Ns$ is the designed number of switching lines for servicing of the offered traffic and $Erl_b(ofr.Ys, Ns)$ is the Erlang formula.

In the recommendation [9], two different methods for evaluation of the offered traffic are presented. The first approach is based on the computational procedure for evaluation of the equivalent offered traffic and the assumption that the offered traffic in the telecommunication system is equal to the equivalent traffic. A drawback of this approach is that repeated attempts are not taken directly into account. In the second approach in the recommendation ITU E.501, the impact of the repeated call attempts is taken into account in the evaluation of the offered traffic.

Presented, tested and analysed are methods for dimensioning, based on the recommendations of ITU for offered traffic, those of CISCO and our method. The different way of evaluation of the offered traffic in the recommendations of ITU and CISCO ([3]) is in the base of the differences between the methods for dimensioning/redimensioning.

The main differences between the studied methods are:

1. The model proposed by us considers the telecommunication system as a whole, not in the separate nodes as it is the case with the other methods.
2. The ways of evaluation of the offered traffic according to the recommendations of ITU [10, 9] and ours [29] are different.

For the purpose of testing of the results (the computational procedure) of the dimensioning/redimensioning in the studied methods we introduce the one new notion – indicative point.

**Definition 2.** *(Indicative point) The point in which the empirical state of the system coincides with the target state, i.e.* $emp.Pbs = trg.Pbs$*, is called indicative point.*

With the prefix *ind* (indicative) we denote the parametric values of the system tuple in the indicative point, for example $ind.Yab, ind.Fa, ind.Ta$ [21].

*Note:* We have proposed an approach to the verification whether one considered method for dimensioning/redimensioning (through which the designed number of switching lines $dsn.Ns$ is determined) leads to correct results (that is to obtain equality) through comparison of $emp.Pbs$ with $Erl_b(emp.ofr.Ys, dsn.Ns)$ in the indicative point. The verification is based on the fact that in the indicative point the designed state of the system has to coincide with the current (empirical) state of the system, i.e. $dsn.crr.Ys = emp.crr.Ys$ and, respectively, $dsn.ofr.Ys = emp.ofr.Ys$, $dsn.Ns = emp.Ns$.

The whole theoretical interval in which the telecommunication system is considered is “divided” by the indicative point into left and right intervals. For each of the methods for dimensioning/redimensioning considered, investigated and compared are the results from [30] :

1. The impact of the assumption for discreteness of the switching lines on the accucarcy of evaluated designed number (the error because of this assumption represents an unavoidable methodical error)[23].
2. The sensibility of the studied methods (the impact of a change in the number of switching lines by one in the whole theoretical interval on the probability for blocking $test.Pbs$)[24].
3. The applicability of each of the methods in the whole theoretical interval on the basis of the introduced criteria for comparison of their applicability.

**Numerical results from the comparison (testing) and their analysis** Numerical experiments are carried out for the comparison of the results. The numerical results are tested. Universal method for quantitative comparison of the numerical results obtained through our method and the widely accepted methods for dimensioning is proposed. Computer program for quantitative comparison (testing) of the methods is developed.

The numerical results from the dimensioning carried out through ITU-1, ITU-2, Cisco-1, Cisco-2, [22, 25] and the proposed by us method are shown in Fig. 4. The parameters of the methods from E.501 are denoted by indices 1 and 2; the parameters of the Cisco 1 and Cisco 2 are denoted by indices 4 and 5; the parameters from the approach based on E.600 are denoted by index 6; the parameters of our method are denoted by *Proposed Method*.

*Advantages* of the proposed method are:

1. Dimensioning with accuracy up to one switching line unlike the other methods in which the dimensioning is inaccurate not only because of the assumption for discreteness;
2. Guarantees the desired level of QoS in more than 98% of the whole theoretical interval unlike the studied widely accepted methods, among which the greatest percentage (Cisco-2) is less than 13.76%.

The better results obtained through our method are due to:

1. The considered model is overall traffic model of telecommunication system;
2. The users’ behaviour is included in detail.

*The advantages of the use* of our method are:

1. In the NDT, in comparison to other known methods, our method gives better results with respect to the QoS;

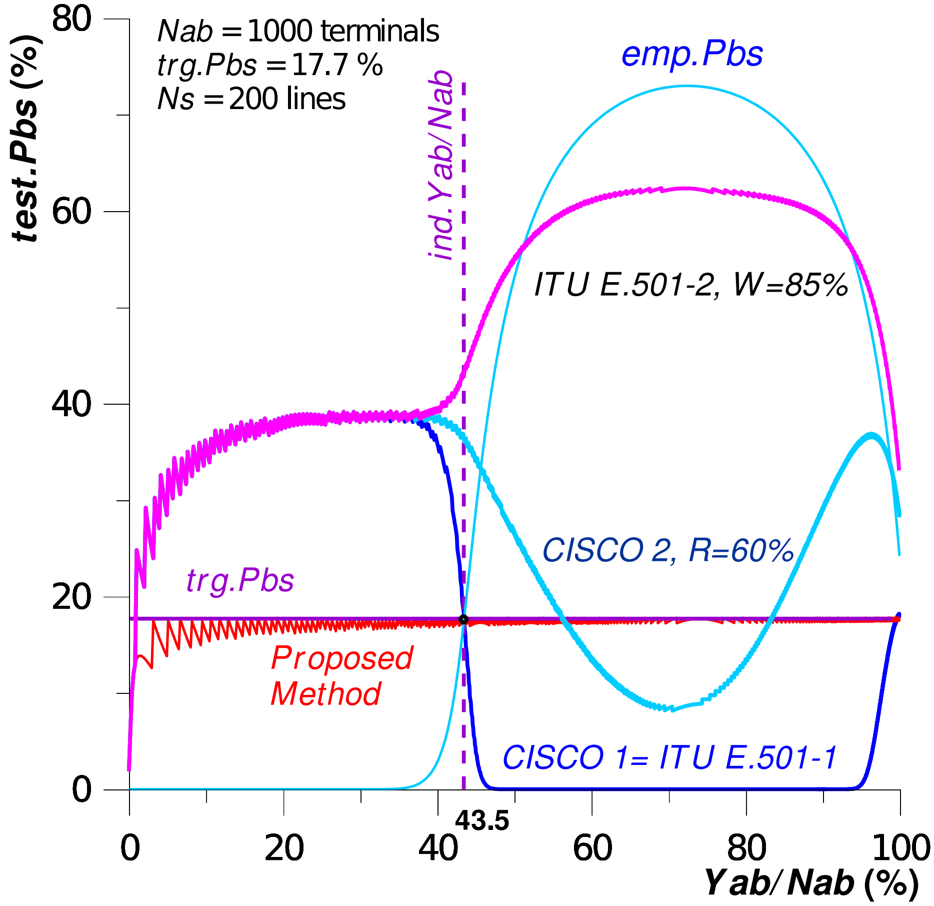


Fig. 4: The probability of blocking due to lack of resources with respect to the normalized designed load of the system $Yab/Nab$. Presented are the test values $test.Pbs$ of ITU-1, ITU-2, Cisco-1, Cisco-2, ITU-2 and the proposed method. The normalized designed load of the system for one terminal of the tuple in the indicative point is denoted by $ind.Yab/Nab$. The target probability of blocking due to lack of resources is $trg.Pbs$ while the empiric is denoted by $emp.Pbs$ [21].

2. In the evaluation of the QoS through the proposed method both the technical characteristics and the users'/customers' attitude are taken into account;
3. Because the proposed method considers overall telecommunication system it allows the construction of analytical expressions for evaluation of the QoE.

One *disadvantage* of the proposed method is that additional measurements are needed in comparison to the other studied methods because it is based on detailed account of the users' behaviour.

The *novelties* in our research on the modeling of networks are:

1. In the conceptual modeling of the systems in the formation of QoS both the technical characteristics and the impact of the human behaviour are taken into account. As an example, in the conceptual modeling a telecommunication network is considered because it is the most widely studied through various methods and allows the comparison of the results obtained through the proposed by us method and the results obtained through other widely accepted methods;
2. The analytical model constructed on the basis of the conceptual model is genuine. It is based on:
   - Easily measurable empiric (primary) parameters, for instance the empiric values of the carried by the service system traffic ($emp.crr.Ys$), the duration of service (time occupancy of the called subscriber $Tb$) etc.;
   - The designed (secondary) parameters, evaluated through analytical expressions based on the empirical, target and the administratively determined such as the empirical values of the input flow from one idle terminal ($emp.Fo$), the target value of the blocking due to lack of resources ($trg.Pbs$) and the administratively set number of subscribers in the service system ($Nab$). Such parameters are, for example, $dsn.Pbr, dsn.Ys$ and $dsn.ofr.Ys$.

     For the proposed analytical model:

     (a) assertions for the existence and uniqueness of the solutions of the systems of equations are proved;

     (b) analytical conditions for existence, uniqueness, and boundary conditions are derived.
3. Formulation and evaluation of the $dsn.ofr.Ys$ which is the base for the evaluation of $dsn.Pbr$ which in turns depends on $emp.Fo$. While $emp.Fo$ is a function of $emp.Pbr, emp.crr.Ys, Tb, emp.Pbs$.

The research can be applied to:

1. Prediction of some parameters (for example the load, the probabilities for blocking etc.), characteristics of the system (efficacy etc.);
2. Management and operational decisions about the work of the system.

# 5 A model of overall telecommunication system including a queuing system in the Switching stage

## 5.1 Conceptual models of queuing systems in service networks

As a continuation of our work on the modeling of overall telecommunication systems, in [28, 40] we have studied the possible representations of queuing systems in Service Systems Theory. Here, we shall present briefly only two of the models studied there – the classical representation of a queuing system and a Generalized Net (GN, see [2]) model corresponding to it.

The classical representation is shown in Fig. 5. In the classical conceptual

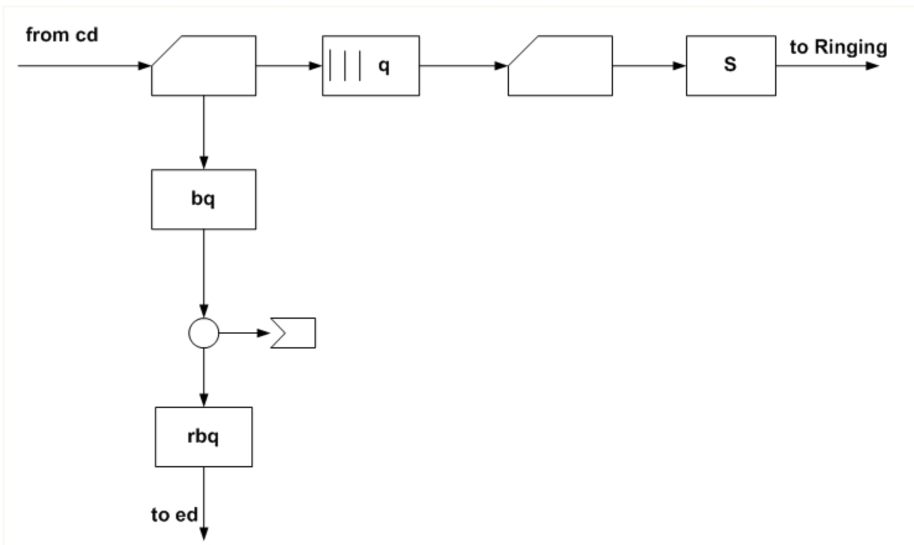


Fig. 5: Conceptual model of a part of the Switching stage of an overall telecommunication system with a queue. **cd** stands for "carried dialing" device, **q** for the Queue device, **s** for "Switching system", **bq** for "blocked queuing", **rbq** for "repeated blocked queuing", **ed** for "enter dialing".

model (see [19]), once the Switching system reaches its capacity, the incoming call attempts are blocked and they are redirected to the "blocked switch" branch which begins at the virtual device denoted by **bs** on Fig. 2. With the inclusion of queue in the Switching stage of the model when the Switching system has reached its capacity the incoming call attempts wait in a buffer until a service line in the Switching system becomes available. We consider the buffer size of the queuing system to be of finite length and the number of servers (service lines) also to be finite. In such queuing system, the call attempts will be blocked only when both the Switching system and the buffer have reached their capacity.

In comparison to the classical conceptual model in Fig. 2, the branch **bs** is removed because the blocked call attempts from the Enter Switch remain in the queue and they are not redirected to other virtual devices. The Switching system with a queue consists of a device of type Queue denoted by **q**, the Enter Switch before it and all devices of the **bq** branch. The switching system is denoted by **s** in Fig. 5. The Enter Switch device before the **q** device redirects the call attempts when the queue is full. The base device **q** has the same parameters as the other base devices: $Fq, Yq, Tq, Pq, Nq$. The capacity of the buffer is $Nq$. The queue discipline considered in the model is FIFO. The Enter switch device between the **q** device and the **s** device has one important parameter – the probability of blocked switching ($Pbs$) with which the call attempts remain in the **q** device.

The classical representation of the queuing systems does not show excplicitly the case when the server has not reached its capacity and the requests are serviced by the buffer device without delay.

A GNs representation of a queuing system is proposed in [28] and it is compared with the conceptual models based on Service Systems Theory. The graphical representation of the GN corresponding to the classical conceptual model of queuing system is shown in Fig. 6.

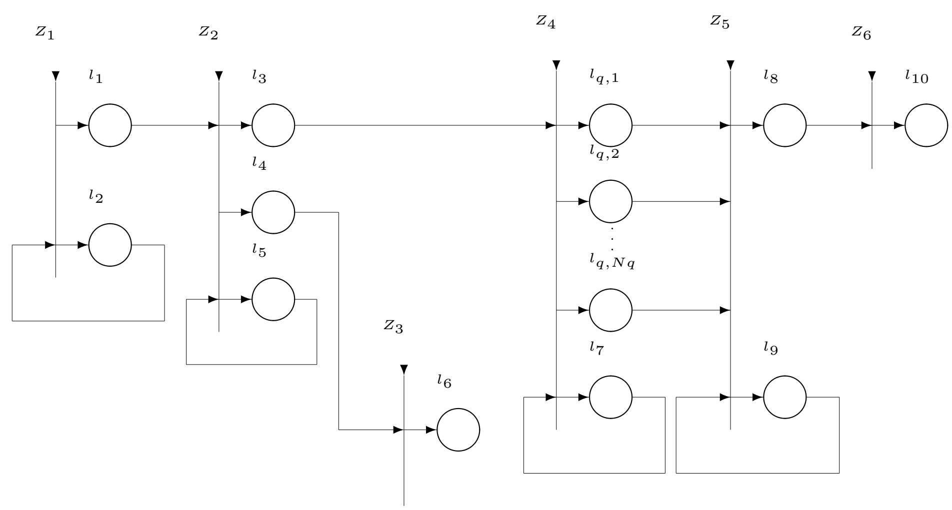


Fig. 6: Generalized net model of a queuing system.

The places of the net correspond to virtual devices in the following way:

- $l_1$ and $l_2$ represent the Generator before Transition 1 in Fig. 5;
- $l_3$ has no analogue in Fig. 5;
- $l_4$ and $l_5$ represent the “blocked queuing” (bq) device in Fig. 5;
- $l_6$ represents the Terminator device after the **bq** device in Fig. 5;
- $l_{q,1}, l_{q,2}, ..., l_{q,Nq}, l_7$ represent the **q** device in Fig. 5;
- $l_8$ and $l_9$ represent the s device (comprising device of the Switching system);

– $l_{10}$ corresponds to the Terminator device after the **s** device in Fig. 5.

Each of the six transitions has the following meaning:

– $Z_1$ represents the function of the Generator before Transition 1 in Fig. 5;
– $Z_2$ represents the function of Transition 1 in Fig. 5;
– $Z_3$ represents the function of the Director between the **bq** device and the Terminator device in Fig. 5;
– $Z_4$ represents the service of the call attempts in the queue;
– $Z_5$ represents the service of call attempts in the Switching system (the s device in Fig. 5);
– $Z_6$ represents the function of the Director between the Switching system and the Terminator in Fig. 5.

Four different types of tokens are used in the model. Detailed description of the transitions can be found in [28].

The conceptual model presented in Fig. 5 gives a clearer connection between the functions of the real system and their visual representations. It allows for easy understanding of the connections between the virtual devices and their functions. A downside of this approach is that it uses 5 different virtual devices. It shows the path of the call attempts in the system but they are not shown in the graphical representation of the model. On the other hand, the GN models use less different components to describe the devices and the paths of the calls: places, arcs, transitions. That is why the GN representations are, in a sense, graphically simpler. However, users need special training in order to understand the paths of the calls and the connections between the analogues of the virtual devices in the net. The comparison of the different conceptual models of queuing systems shows that the GN representation is less suitable for the construction of analytical models of overall telecommunication systems.

### 5.2 Conceptual model of overall telecommunication system including a queuing system

The first conceptual model of overall telecommunication system including a queuing system in the switching stage is described in [1]. In this model the classical representation of queuing systems is used. Here we propose a detailization of the model, and in particular a more detailed representation of the queuing system which makes the model easier to understand and more suitable for the derivation of equations for the parameters of the queuing system. The graphical representation of the model is shown in Fig. 7.

In the conceptual model in Fig. 7 there are at least 39 important virtual devices. Of them 34 are base virtual devices and 4 (**a, b, s, ab, w**) are comprising. They are of interest because the values of their parameters characterize the state of the overall telecommunication system. Every device has five parameters: $P, F, T, Y$ and $N$. Therefore the total number of parameters is 195.

As in the classical model described in Section 2, the names of the base virtual device are formed as a concatenation of the first letters of the branch exit, branch

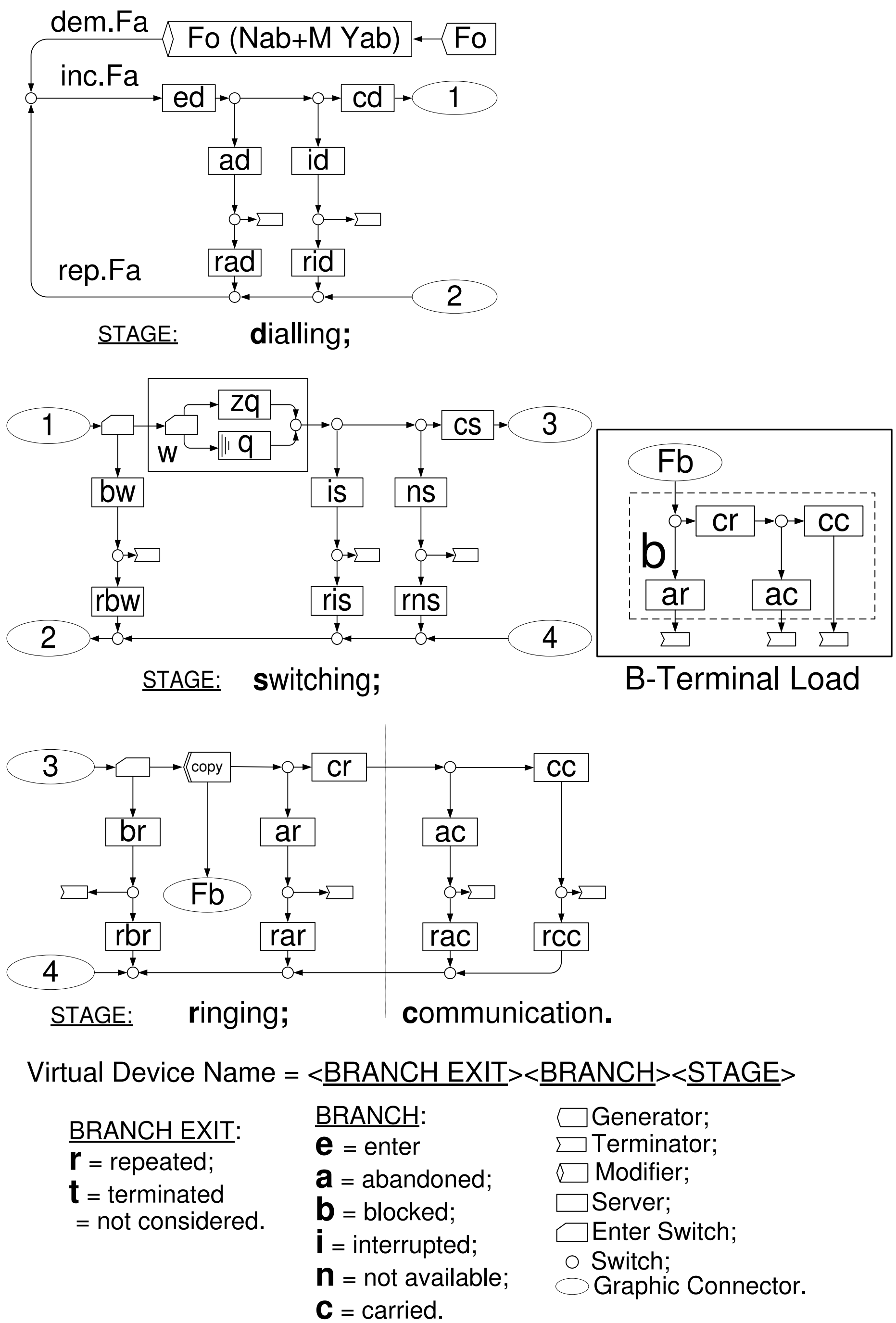


Fig. 7: Conceptual model of overall telecommunication system including a queueing system in the switching stage.

and the stage in which the device is situated. The only exception to this are the **bw** and **rbw** devices which stand for "blocked waiting" and "repeated blocked waiting" respectively. The comprise virtual devices with the exception of the **w** device are the same as in the classical model. The inclusion of the **w** device whose name comes from "waiting" allows for a more detailed representation of the queuing system in comparison to the model in Fig. 5. If both the **w** and the **s** device (not shown on the figure) have reached their capacity, the requests are sent to the *bw* device. The corresponding probability of this to happen is denoted by $Pbw$. If there are free places in the $w$ device, the requests enter the Enter switch device inside the $w$ device. This happens with probability $1 - Pbw$. From there the requests are serviced in two ways - with waiting or without waiting – depending on the number of requests in the switching system ($Ys$), as follows.

- If the switching system has reached its capacity, the requests enter the **q** device. The corresponding probability of this to happen is denoted by $Pq$.
- If the switching system has not reached its capacity, the requests enter the **zq** device. The corresponding probability of this to happen is denoted by $Pzq$.

We shall consider that the mean service time of the requests in the **zq** device $Tzq$ is equal to 0. In this way the mean service time of the requests in the **w** device can be expressed in the following way

$$Tw = PqTq + (1 - Pq)Tzq = PqTq\,. \tag{58}$$

Similarly to [1], a base tuple of parameters can be determined. The parameters of the base tuple for the present conceptual model may be divided into two groups as follows:

- Static parameters: $M', Nab, Ns, Ted, Pad, Tad, Prad, Pid, Tid, Prid, Ted, Pis, Tis, Pris, Pns, Tns, Tes, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc, Nq, Tbw, Trbw, Prbw$. Their values are considered independent of the system state $Yab$ (see [21]) but may depend on other factors. For the model time interval they are considered constants.
- Dynamic parameters: $Yab, Fa, Pbw, Tw, Pbr, Pq, dem.Fa, rep.Fa, ofr.Fw, crr.Fs, T_s, F_s, T_{cs}, Y_s, Yw$. Their values are mutually dependent. Equations expressing their dependencies can be derived with the help of the graphical representation of the conceptual model in Fig. 7.

The parameters can be also classified on the basis of the origin of their values.

- Parameters related to the technical characteristics of the system: $Pid, Pis, Tcs, Ns, Nw = Nq$;
- Parameters describing the human behaviour: $Fo, Nab, Prad, Tid, Prid, Pris, Tis, Pns, Tns, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Prac, Tcc, Prcc, Tbw, Trbw, Prbw$;
- Mix factors' parameters: $Ted, Pad, Tad, Tcd, Pac, Tac$. They are dependent on the first two groups;

- Parameters whose value is determined by the modellers: $M'$. It characterizes a Bernoulli-Poisson-Pascal (BPP) flow;
- Parameters derived from the previous groups: $Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fw, crr.Fs, Tw, Pbw$.

The output (dynamic) parameters characterizing the Quality of Service (QoS) are $Pbr, Pbs, Tw$.

This classification of the parameters allows for different types of teletrafic tasks to be formulated and solved.

In order to be able to derive relatively simple equations for the dynamic parameters, as in the classical model described in Section 2, we should make some assumptions. For the present model, all assumptions about the classical model hold. There is a difference only in the following assumptions:

A-1* The telecommunication system considered is represented graphically and functionally in Fig. 7 and it is closed.

A-2* (Device Capacity) All base virtual devices in the model, except the **q** device, have unlimited capacity. The **q** device has capacity $Nq$. Comprising devices have limited capacities: **ab** device contains all the active terminals; switching system (**s**) has capacity of $Ns$ calls (every internal switching line may carry only one call); the **w** device has finite capacity - $Nw$; every terminal has capacity of one call, common for both incoming and outgoing calls.

### 5.3 Derivation of analytical expressions for the parameters of the queuing system

We consider the conceptual model of overall telecommunication system with queue shown in Fig. 7 and described in Section 2. Parameters with known values are: all probabilites for directing the call to a device (the P–parameters), with the exception of $Pbw, Pq, Pbr$; the holding time parameters of the base virtual devices (T – parameters), except $Tq$ and $Tcs$. The unknown parameters are all dynamic parameters.

We want to express analytically the unknown parameters' values of the **w** device: waiting time ($Tw$), probability of blocked waiting ($Pbw$), length of the queue ($Yw$).

In order to compactly describe single queuing stations in an unambiguous way, the so called Kendall notation is often used (see [4]). A queuing system is described by 6 identifiers separated by vertical bars in the following way:

*Arrivals/ Services/ Servers / Buffersize / Population / Scheduling*

where "Arrivals" characterises the arrival process (arrival distribution), "Service" characterizes the service process (service distribution), "Servers" – the number of servers, "Buffersize" – the total capacity, which includes the customers possibly in the server (infinite if not specified), "Population" – the size of the customer population (infinite if not specified), and finally, "Scheduling" – the employed service discipline.

In our model, the queuing system in the Switching stage of the telecommunication network in Kendall notation is represented as $M|M|Ns|Ns+Nw|Nab|FIFO$, where $M$ stands for exponential distribution, $Ns$ is the capacity of the Switching system (number of equivalent internal switching lines) and $Nab$ is the total number of active terminals which can be calling and called. This is related to the derivation of the analytical model of the system.

The queueing system in the switching stage differs from the queuing systems studied in [38, 39] in that it has more exits. The exits are represented in the conceptual model in Fig. 7 with the branches $is$ (interrupted switching), $ns$ (not available switching), $br$ (blocked ringing), $ac$ (abandoned communication), $cc$ (carried communication). In [1], we have derived analytical expressions for the parameters of the queuing system, starting with the simplest queuing system $M|M|1|FIFO$ and gradually advancing to the most complicated system with finite buffer and finite capacity of the server. Here we shall use the results from [1] but adapted to the more detailed conceptual model presented here.

The density functions of the arrival and service times are respectively

$$a(t) = \lambda e^{-\lambda t}\,, \tag{59}$$

$$b(t) = \mu e^{-\mu t}\,, \tag{60}$$

where $1/\lambda$ is the mean value of time between two arrivals (interrarival time) and $1/\mu$ is the mean time of service. For our queuing system, $\lambda = ofr.Fw$ and $\mu = Fs$. They are assumed to be statistically independent which results in a birth-death process. Let us denote with $p_n$ the probability that the queuing system is in state $n$ that is

$$p_n = Pr\{\text{there are n call attempts in the queuing system}\}.$$

There are different ways to solve the birth-death equations. The solution is well-known and can be found for example in [38]. First, we notice that the arrival rate $\lambda_n$ is equal to 0 when $n \geq Ns + Nw$. The probability for the system to be in state $n$ is now given by

$$p_n = \begin{cases} \frac{(ofr.Fw)^n}{n!(Fs)^n} p_0 & \text{for } 1 \leq n < Ns. \\ \frac{(ofr.Fw)^n}{(Ns)^{n-Ns} Ns!(Fs)^n} p_0 & \text{for } Ns \leq n \leq Ns + Nw. \end{cases} \tag{61}$$

Again, the condition that the sum of the probabilities $p_n$ should be equal to 1, gives us the following expression for $p_0$:

$$p_0 = \left( \sum_{n=0}^{Ns-1} \frac{(ofr.Fw)^n}{n!(Fs)^n} + \sum_{n=Ns}^{Ns+Nw} \frac{(ofr.Fw)^n}{(Ns)^{n-Ns} Ns!(Fs)^n} \right)^{-1} . \tag{62}$$

In order to simplify the expression we set $r = ofr.Fw/Fs$ and $\rho = r/Ns$. After elementary operations above expression for $p_0$ becomes

$$p_0^{-1} = \begin{cases} \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} \frac{1-\rho^{Nw+1}}{1-\rho} & \text{for } \rho \neq 1 \,. \\ \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!}(Nw+1) & \text{for } \rho = 1 \,. \end{cases} \tag{63}$$

Using the above, we derive analytical expressions about the important parameters: mean waiting time for all requests ($Tw$), probability of blocked waiting ($Pbw$) and expected length of the queue ($Yw$).

The probability of blocked waiting ($Pbw$) is equal to the probability that the system is in state $Ns + Nw$ and from (61) we have

$$Pbw = \frac{(ofr.Fw)^{Ns+Nw}}{(Ns)^{Nw} Ns!(Fs)^{Ns+Nw}} p_0 \,. \tag{64}$$

For the expected length of the queue in this case which is also equal to $Yw$ we have

$$Yw = \sum_{n=Ns+1}^{Ns+Nw} (n-Ns)p_n = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(\rho-1)\rho^{Nw}(Nw+1)+1-\rho^{Nw+1}] \,. \tag{65}$$

The mean service time of the requests in **w** device for both the waiting and non-waiting requests, given the condition $Tzq = 0$, is

$$Tw = Pq\,Tq + (1-Pq)Tzq = Pq\,Tq \,. \tag{66}$$

The mean service time in the **q** device ($Tq$) is given by

$$Tq = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} \frac{[(\rho-1)\rho^{Nq}(Nq+1)+1-\rho^{Nq+1}]}{ofr.Fw(1-Pbw)} \,. \tag{67}$$

The probability $Pq$ is the probability that the system is in any of the states $Ns, Ns+1, ..., Ns+Nw-1$, i.e.

$$Pq = \sum_{k=Ns}^{Ns+Nw-1} p_k = \sum_{k=Ns}^{Ns+Nw-1} \frac{(ofr.Fw)^k}{(Ns)^{k-Ns} Ns!(Fs)^k} p_0 \,. \tag{68}$$

After simpification we obtain

$$Pq = \frac{p_0 (Ns)^{Ns} \rho^{Ns} (1-\rho^{Nw})}{Ns!(1-\rho)} \,. \tag{69}$$

After substitution of (69) and (67) in (66) we obtain

$$Tw = \frac{p_0^2 (Ns\rho r)^{Ns} \rho (1-\rho^{Nw})}{(Ns!)^2 (1-\rho)^3} . \frac{[(\rho-1)\rho^{Nw}(Nw+1)+1-\rho^{Nw+1}]}{ofr.Fw(1-Pbw)} \,. \tag{70}$$

## 6 Directions for future research

In the present paper, new methods for conceptual and analytical modeling of overall telecommunication systems are presented including new results regarding overall telecommunication systems with queueing. They have been developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. The use of these models gives better accuracy in comparison to the other methods proposed in the scientific literature. Some directions for future research include:

- determining the degree to which the use of GNs makes the analytical modelling of overall telecommunication system easier;
- determining whether the classical teletraffic theory is suitable for the modeling of overall telecommunication systems.
- use of the new results for prediction of the QoE in overall telecommunication systems.

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